Cosmology with a Variable Chaplygin Gas

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Abstract

We consider a new generalized Chaplygin gas model that includes the original Chaplygin gas model as a special case. In such a model the generalized Chaplygin gas evolves as from dust to quiessence or phantom. We show that the background evolution for the model is equivalent to that for a coupled dark energy model with dark matter. The constraints from the current type Ia supernova data favour a phantom-like Chaplygin gas model.

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1 Introduction

Recent observations of type Ia supernovae (SNe Ia) suggest that the expansion of the universe is accelerating and that two-thirds of the total energy density exists in a dark energy component with negative pressure [1]. In addition, measurements of the cosmic microwave background (CMB) [2] and the galaxy power spectrum [3] also indicate the existence of the dark energy. The simplest candidate for the dark energy is a cosmological constant Λ, which has pressure $P_\Lambda = -\rho_\Lambda$. Specifically, a reliable model should explain why the present amount of the dark energy is so small compared with the fundamental scale (fine-tuning problem) and why it is comparable with the critical density today (coincidence problem). The cosmological constant suffers from both these problems. One possible approach to constructing a viable model for dark energy is to associate it with a slowly evolving and spatially homogeneous scalar field ϕ, called "quintessence" [4, 5]. Such a model for a broad class of potentials can give the energy density converging to its present value for a wide set of initial conditions in the past and possess tracker behavior (see, e.g., [6] for reviews with more complete lists of references).

However, neither dark matter nor dark energy has laboratory evidence for its existence directly. In this sense, our cosmology depends on two untested entities. It would be nice if a unified dark matter/energy (UDME) scenario can be found in which these two dark components are different manifestations of a single cosmic fluid [7, 8]. An attractive feature of these models is that such an approach naturally solves, at least phenomenologically, the coincidence problem. As a candidate of the UDME scenarios, the Chaplygin gas model was recently was proposed [9]. The Chaplygin gas is characterized by an exotic equation of state $P = -A/\rho$, where A is a positive constant. Such equation of state leads to a component which behaves as dust at early stage and as cosmological constant at later stage. The Chaplygin gas emerges from the dynamics of a generalized d-brane in a $(d+1, 1)$ spacetime and can be described by a complex scalar field whose action can be written as a generalized Born-Infeld action [10]. The model parameters were constrained using various cosmological observations, such as SN Ia data [11], CMB experiments [12] and other observational data [13]. The Chaplygin gas model has been extensively studied in the literature [14].

Recently, there are some indications that a strongly negative equation of state, $w \leq -1$, may give a good fit [15]. Here we propose a new generalized Chaplygin gas model in which the Chaplygin gas can act like either quiessence with $w > -1$ or phantom with $w < -1$ at low densities. Such a generalized model may formally be derived from a Born-Infled Lagrangian density for a scalar field. Alternatively, the same background evolution may arise if there is an interaction between dark energy and dark matter. We analyze its cosmological consequences and then constrain the parameters associated with the generalized Chaplygin gas using the recent SN Ia data.

1

2 Model

Let us now consider a Born-Infeld Lagrangian [16]

$$\mathcal{L}_{\mathrm{BI}} = V(\phi)\sqrt{1 + g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi}, \tag{1}$$

Where $V(\phi)$ is the scalar potential. In a spatially flat Friedmann-Robertson-Walker (FRW) universe, the energy density and the pressure are given by $\rho = V(\phi)(1 - \dot{\phi}^2)^{-1/2}$ and $P = -V(\phi)(1 - \dot{\phi}^2)^{1/2}$, respectively. The corresponding equation of state is formally given by

$$P = -\frac{V^2(\phi)}{\rho}. \tag{2}$$

If one rewrites the self-interaction potential as a function of the cosmic scale factor: $V^2(\phi) = A(a)$ [17], then, by inserting Eq. (2) into the the energy conservation equation, $d\rho/d\ln a = -3(\rho + P)$, one finds that the energy density evolves as

$$\rho = a^{-3}\left[6\int A(a)a^5 da + B\right]^{1/2}, \tag{3}$$

where B is an integration constant. Given a function $A(a)$, Eq. (3) allows us to obtain a solution $\rho(a)$ in principle. Following Refs. [17] we set $A(a) = A_0 a^{-n}$ where A_0 and n are constants, and where we take $A_0 > 0$ and $n < 4$. By taking explicitly the integral (3) it follows that

$$\rho = \left[\frac{6}{6-n}\frac{A_0}{a^n} + \frac{B}{a^6}\right]^{1/2}. \tag{4}$$

We find that $n = 0$ corresponds the original Chaplygin gas model which interpolates between a universe dominated by dust and a De Sitter one. Compared to the original Chaplygin gas, in this generalized model (called as variable Chaplygin gas) the universe tends to be a quiessence-dominated ($n > 0$) [18] or phantom-dominated one ($n < 0$) [19] with constant equation of state parameter $w = -1 + n/6$. The first term on the right hand side of Eq. (4) is initially negligible so that the expression (4) can approximately be written as $\rho \sim a^{-3}$, which corresponds to a universe dominated by dust-like matter. Once the first term dominates, it causes the universe to accelerate. In this case we find $a \sim t^{4/n}$ so that the expansion is accelerated for $n < 4$. Defining

$$B_s \equiv \frac{B}{6A_0/(6-n) + B}, \tag{5}$$

In a flat FRW universe the Hubble parameter is now given by

$$H(z) = H_0\left[B_s(1+z)^6 + (1-B_s)(1+z)^n\right]^{1/4}, \tag{6}$$

where $z = 1/a - 1$ is redshift and H_0 is the present value of the Hubble parameter. There are two free parameters in this model, B_s and n.

It is easy shown that the background evolution for the variable Chaplygin gas model is equivalent to that for an interaction model between the dark matter and the dark energy with $w = -1 + n/6$ [20]. Assuming the scaling behaviour for the dark energy density and the dark matter density, $\rho_x \propto \rho_m a^{6-n}$, in a flat FRW universe it is straightforward to get

$$\rho_m + \rho_x = \rho_0 \left[(1 - \Omega_{m0})a^{-n} + \Omega_{m0}a^{-6}\right]^{1/2}, \tag{7}$$

where ρ_0 is the critical density and Ω_{m0} is the matter density parameter. If the coupled system can be written as $\dot{\rho}_m + 3H\rho_m = Q$ and $\dot{\rho}_x + nH\rho_x/2 = -Q$, such scaling solutions follow from an interaction characterized by [21]

$$Q = -3H\frac{1 - n/6}{1 + \rho_m/\rho_x}\rho_m, \tag{8}$$

which indicates that there is a continuous transfer of energy from the dark matter component to the dark energy for $n < 4$. Comparing Eq. (7) with Eq. (6) we see that B_s can be interpreted as an effective matter density. In the interaction scenario, constraints on the model parameters $(\delta_0, w_\phi, \Omega_{m0})$ from the SN data have been derived in Ref. [22]. The variable Chaplygin gas model corresponds to an interaction case with $\delta_0 = 3(1 - \Omega_{m0})w_\phi$, which corresponds to a line on the (δ_0, w_ϕ) plane given a value of Ω_{m0}. Form Fig. 6 in Ref. [22], we see that our model is consistent with their constrains on the interaction model.

3   SN Ia Constraints

We now consider constraints on the model through a statistical analysis involving the most recent SN Ia data, as provided recently by Riess *et al.* [23]. The total sample presented in Ref. [23] consists of 186 events distributed over the redshift interval $0.01 \leq z \leq 1.7$ and constitutes the compilation of the best observations made so far by the two supernova search teams plus 16 new events observed by the Hubble Space Telescope. This total data set was divided into gold and silver subsets. Here we will consider only the 157 events that constitute the so-called gold sample.

The parameters in the model are determined by minimizing

$$\chi^2(H_0, B_s, n) = \sum_i \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{mod}}(z_i; H_0, B_s, n)]^2}{\sigma_i^2}, \tag{9}$$

where σ_i is the total uncertainty in the observation, the distance modulus $\mu(z_i)$ is

$$\mu(z_i) = 5\log_{10}\frac{d_L(z_i)}{\text{Mpc}} + 25, \tag{10}$$

and the luminosity distance in the spatially flat FRW model with variable Chaplygin gas is given by

$$d_L = cH_0^{-1}(1 + z)\int_0^z dz \left[B_s(1 + z)^6 + (1 - B_s)(1 + z)^n\right]^{-1/4}. \tag{11}$$
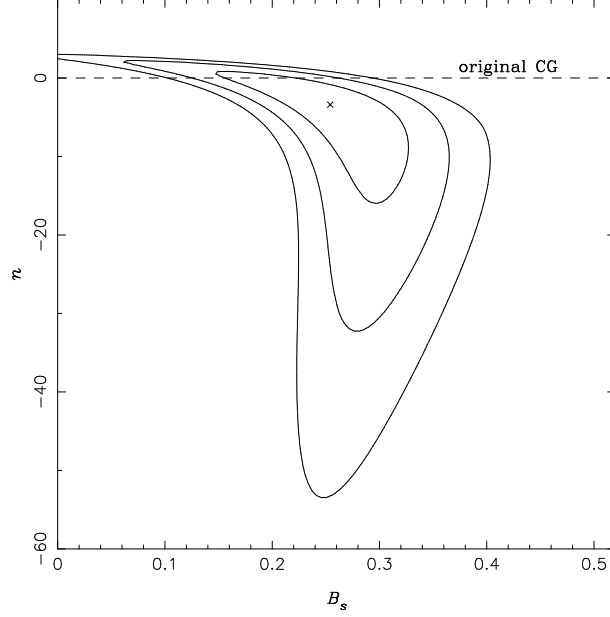
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Figure 1: Probability contours at 68.3%, 95.4% and 99.7% confidence levels for B_s versus n in the variable Chaplygin gas model from the gold sample of 157 SN Ia data. The dashed line represents the original Chaplygin gas model with $n = 0$. The best fit happens at $Bs = 0.25$ and $n = -3.4$.

To determine the likelihood of the parameters B_s and n, we marginalize the likelihood function $L = \exp(-\chi^2/2)$ over H_0. We adopt a Gaussian prior $H_0 = 72 \pm 8\text{km s}^{-1} \text{ Mpc}^{-1}$ from the Hubble Space Telescope Key Project [24]. The results of our analysis are displayed in Fig. 1. The best fit of the model gives that $B_s = 0.25$ and $n = -3.4$ with $\chi^2 = 174.54$. The three contours correspond to 68.3%, 95.4% and 99.73% confidence levels, respectively. We can see that current SN Ia constraints fovour a phantom-like Chaplygin gas model.

The age of the universe, $t(z)$, and the deceleration parameter, $q(z)$, are given by

$$t(z) = \int_z^\infty \frac{dx}{(1+x)H(x)}, \tag{12}$$

$$q(z) = \frac{d \ln H(z)}{d \ln(1+z)} - 1. \tag{13}$$

Fig. 2 shows the evolution of the age of the universe with redshift. We find that the best-fit age of the universe today is $t_0 = 12.3$ Gyrs if the Hubble parameter is taken to be $H_0 = 72$ km s^{-1} Mpc^{-1} [24], which is slightly lower than the age of a ΛCDM universe, $t_0 = 13.4$ Gyrs. This age estimate is consistent with the results, $t_0 = 12.5 \pm 2.5$ Gyrs at 95% confidence level, from the oldest globular clusters [25].

Fig. 3 shows the evolution of the deceleration parameter with redshift. We find that
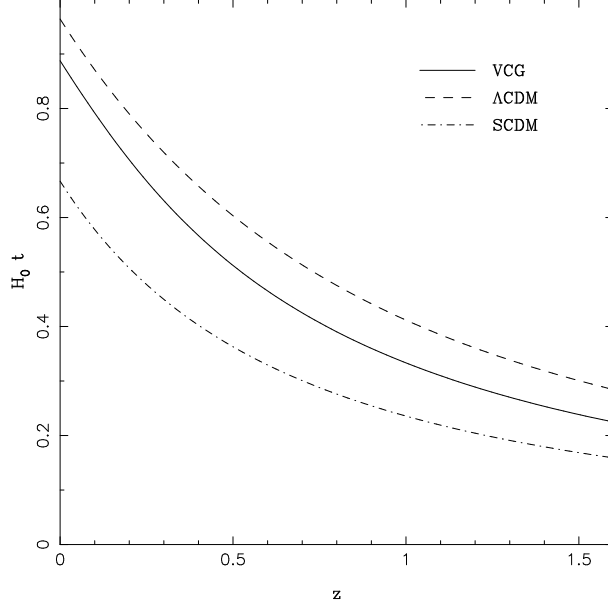
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Figure 2: The age of the universe, $H_0 t(z)$, is shown as a function of the redshift for the variable Chaplygin gas model with $Bs = 0.25$ and $n = -3.4$ (solid line), ΛCDM with $\Omega_{m0} = 0.3$ and $\Omega_\Lambda = 0.7$ (dashed line) and SCDM (dot-dashed line).

the behaviour of the deceleration parameter for the best-fit universe is quite different from that in the ΛCDM cosmology. The present value of the best-fit deceleration parameter, $q_0 = -1.26$, is significantly lower than $q_0 = -0.55$ for the ΛCDM model with $\Omega_{m0} = 0.3$. Furthermore, the rise of $q(z)$ with redshift is much steeper in the case of the best-fit model, with the result that the universe begins to accelerate at a comparatively lower redshift $z = 0.3$ (compared with $z = 0.7$ for ΛCDM).

4   Conclusions

In this paper, we have considered a new generalized Chaplygin gas model. In this scenario, the variable Chaplygin gas can drive the universe from a non-relativistic matter dominated phase to an accelerated expansion phase, behaving like dust-like matter in early times and as quiessence/phantom in a recent epoch. We have shown that the variable Chaplygin gas model is equivalent to an interaction model between dark energy and dark matter in the sense of the background evolution. Cosmic late-time acceleration implies that there exists a continuous transfer of energy from the dark matter component to the dark energy one. We constrained the parameters associated with the variable Chaplygin gas using the Gold SN sample. We find that the constrains from the SN data favour a phantom-like Chaplygin gas model. This model deserves further investigation as a viable cosmological model.
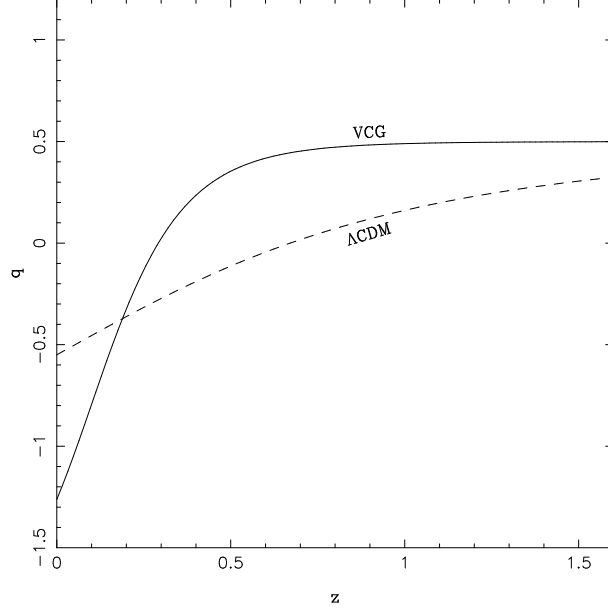
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Figure 3: Evolution of the deceleration of the universe, $q(z)$ with redshift for the variable Chaplygin gas model with $Bs = 0.25$ and $n = -3.4$ (solid line) and ΛCDM model with $\Omega_{m0} = 0.3$ and $\Omega_{\Lambda} = 0.7$ (dashed line).

Acknowledgements

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